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Technical Report

Creation and Dissemination of Upper-elementary Mathematics Assessment Modules

National Science Foundation Math and Science Partnership Program (NSF 08-525)

2009-2012

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Creation and Dissemination of Upper-elementary Mathematics Assessment Modules

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This document provides information on the grades 4 and 5 mathematics test modules developed as a joint Harvard/ETS venture from 2009 to 2012. Hundreds of items that were initially developed were then culled by a series of reviews, revisions, pilot tests, cognitive interviews, and field tests. The result of these efforts yielded eight final test modules that are intended to be used to help measure gains resulting from teacher professional development, and also to be sensitive to variation in teachers' mathematical knowledge for teaching and instruction. Importantly, these modules are aligned with the grade 4 and grade 5 Common Core mathematics standards[1], and can thus reveal important patterns in student achievement with respect to those standards.

The first section of this report describes the item development and pilot design phase of the project. The next section describes the actual piloting, cognitive interviews, and test construction phases that followed item development. The final section includes the psychometric analyses conducted on the eight test forms implemented in the Spring 2012 administration, including (a) classical item statistics, (b) item calibration and equating using the two-parameter item response theory (IRT) model (i.e., generalized partial credit model, GPCM; Muraki, 1992), and (c) the creation of the raw score to scale score conversion table for each test form.

Item Development and Pilot Design

The items described in this report were written between 2009 and 2011. The development of the items contained in the eight test modules was guided by three content-specific advisory panels (Number and Operations, Algebra, and Geometry/Measurement). The advisory panels each met for two days (at separate times) in Cambridge, Massachusetts to discuss relevant research related to the specific content, to help define appropriate developmental models for student understanding, and to create exemplar items to be used to facilitate the development of many more items by ETS assessment specialists. In designing test items, there were five initial goals. Specifically, items should:

- represent *central ideas* in the subject matter;
- focus on the *meaning* of facts and procedures;
- allow accurate placement of students on developmental maps;
- require more complex responses than traditional multiple-choice problems; and
- be written in a "Plain English" style in order to increase accessibility to a wider range of students.

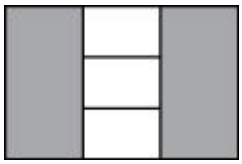
[1] For more information about the Common Core State Standards for Mathematics (CCSSM), go to the following website: http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

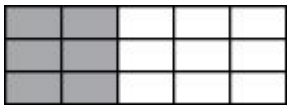
One of the more challenging aspects of this project was designing machine-scorable items for a pencil-and-paper format that would still allow educators to assess the level of understanding a student has for the content being assessed. Out of this challenge came the idea to present "nested sets" of items that would be scored using a rubric that is tied to the development model of the particular content. In other words, students would respond to a set of items, and the set would be scored as a single item (often a multiple-point item) rather than as discrete items. An example of this type of "nested set" is shown in Figure 1.

Figure 1. (Grade 4, Form G)

For numbers 21–24, state whether or not each figure has $\frac{2}{5}$ of its whole shaded.

21.  **(Y)** Yes **(N)** No

22.  **(Y)** Yes **(N)** No

23.  **(Y)** Yes **(N)** No

24.  **(Y)** Yes **(N)** No

This item is more complex than a traditional multiple-choice item in that a student has to evaluate each part separately and decide whether the fraction 2/5 can take different forms. Because two points are assigned to this problem, the item can also provide feedback at the item level as to the depth of understanding a student has about simple fractions. The total number of ways to respond to this item is sixteen. "Guessing" the correct combination of responses is much less likely than it would be for a traditional four-option multiple-choice item. The correct response for this item will receive 2 points, and the points will be earned based on the level of understanding the student has demonstrated. The scoring rubric used for this item follows in Figure 2.

Figure 2.

Scoring Rubric

Responses to this item will receive 0-2 points, based upon the following:

2 points: YNYN The student has a solid understanding of 2/5 as well as an equivalent form of 2/5.

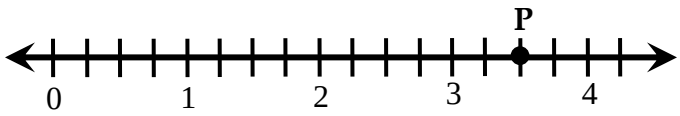
1 point: YNNN, YYNN, YYYN The student has only a basic understanding of 2/5. Either the student doesn't recognize an equivalent fraction for 2/5 or doesn't understand that all 5 parts must be equal-sized in item #23.

0 points: YYYY, YNNY, NNNN, NNYY, NYYN, NYNN, NYYY, YYNY, YNYY, NYNY, NNYN, NNNY The student demonstrates inconsistent understanding of 2/5 or answers "Y" to item #25, clearly showing a misunderstanding of what 2/5 means. Figure #25 is considered a "disqualifier," and an answer of "Y" to this part of the item would cancel out any other correct responses as "guesses" on the part of the student.

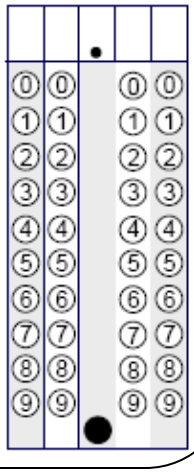
Not all of the items in the eight test modules are in "nested sets" like the example shown above. Some items are indeed traditional multiple-choice items, partly because they are familiar to students and partly because the concepts being assessed didn't justify assigning a value of more than a single point. Another type of item used in these modules is a gridded-response item. The student essentially has to determine the solution and then grid-in a series of bubbles to show his/her answer. An example of this type of "gridded-response" item is shown in Figure 3.

Figure 3. (Grade 5, Form M)

27. **Look at the following number line.**



What decimal number is represented by point P ?



The student responds to this type of item by writing the answer (in this case, 3.5) in the boxes at the top of the grid and then filling in the corresponding circles below each number. It should be noted that while the intent was to make the answer document a truly scannable document, all

documents were actually read and reported by a key entry vendor (100% double reads). The instruction given to the vendor was to record the bubbled-in response unless none existed. If the student wrote the correct response in the boxes above the grid but did not mark the corresponding circles, the student's response was still recorded.

Once the content, types of items, and developmental models were decided upon, ETS assessment specialists proceeded to write, review, and revise items. Harvard educators, as well as advisory panel members, provided additional guidance and reviews throughout the development cycle.

When this project began in 2009, the Common Core State Standards for Mathematics (CCSSM) were not yet available. Therefore, the NCTM focal points were used to determine the content in each of the three domains. After the CCSSM were released in 2010, another review of the items already developed was conducted, and only those items that were also aligned to the new Common Core standards made it into the final test modules.

Piloting, Cognitive Interviews, and Test Construction

In June of 2009, the first of three advisory panels was convened to discuss topics related to Algebra. Several developmental models were discussed, and prototype items were reviewed. It was agreed at the first meeting that items/tasks should be designed to:

- primarily assess one strand;
- be contextualized where appropriate;
- gather evidence from multiple developmental levels;
- employ a set of questions that relate to one another; and
- develop, where possible, the idea of fluency within a strand.

Members of the Algebra panel[2] also suggested items from their own research. Immediately following the two-day meeting, work began on creating a bank of items measuring algebraic concepts.

In August of 2009, the second advisory panel was convened to discuss topics related to Number and Operations. The panel worked to define levels of development and competency/fluency with respect to multi-digit multiplication and division of whole numbers. Additionally, models and prototype items concerning rational numbers (decimals, fractions, and percents) were reviewed, and suggestions were made to pursue the development of items involving addition and subtraction of rational numbers, as well as various representations of rational numbers. As with the Algebra panel, members of the Number and Operations advisory group[3] were asked to create prototype items from their own research. Again, work began immediately following this two-day meeting to create a bank of items measuring the Number and Operations domain.

From its inception, the ultimate goal of this project was to produce a set of test modules that could be used to gauge growth in three mathematical domains: Algebra, Number/Operations, and

[2] Maria Blanton, Bárbara M. Brizuela, Al Cuoco, Eric Knuth
[3] Mindy S. Kalchman, Dick Lesh, Ed Silver, Tad Watanabe

Geometry/Measurement. However, in order to take the novel approach to assess students' knowledge more deeply than traditional multiple-choice items generally allow, and still use a machine-scorable format, the developers decided to move forward by creating five initial test modules that included a mix of Number/Operations items and Algebra items. As noted below, we reaffirmed this decision over the next several years of assessment development and piloting, meaning that the eight forms we described above contain a mix of these three topic areas, and report only one overall scale score for each student.

The initial five forms were piloted with nearly 200 students (93 from grade 4 and 99 from grade 5) at Florida State University School (FSUS). FSUS is a K-12 research school with a student population demographically controlled to mirror the population of the entire state of Florida. By choosing this school for pilot testing, a good balance of gender, racial/ethnic background, and intellectual abilities[4] of students was obtained. The following information provides some specifics of this first round of pilot testing:

- All five test modules were randomly administered to both grade 4 and grade 5 students.
- The number of items on these modules ranged from a low of 34 to a high of 45.
- All test modules were administered in the students' classrooms by co-PI Judy Hickman over a period of one week.
- A series of sample items[5] was given to students immediately prior to the administration of the test modules.
- Only eight students failed to complete the test modules in 45 minutes or less. Students were allowed to leave items blank if they did not understand how to respond.
- Ten fourth-grade students and ten fifth-grade students were individually interviewed following the administration of the test modules (within 1 day) to gain insight into students' thinking about the items.
- All students were encouraged to make comments on their test booklets concerning items they liked, didn't like, didn't understand, were confused about, and so forth. All comments were compiled and reviewed.

Following this initial round of pilot testing, items were revised and more items were written to fill in gaps in development for the Number/Operations and Algebra domains. The next phase involved moderate-scale field testing of test modules that followed a specific design. Four test modules (Forms A, B, C, D) were constructed that were each 50 items[6] in length. Items assessing Number/Operations concepts made up roughly 60% of each form, while items assessing Algebra concepts made up the remaining 40% of each form.

Several elementary schools in Leon County, Florida, were selected for field testing. Again, co-PI Judy Hickman administered all the field tests over a two-week period in the Spring of 2010. Having a single test administrator helped to ensure consistency in the conditions in which

[4] IQ testing is administered to all enrollees at FSUS in order to control for intellectual variance.
[5] Sample items were necessary to give students the opportunity to ask about how to respond to both grid-in type items and "nested" item sets in which more than one answer could be correct.
[6] While the sequencing on these tests indicates 50 items, the scoring will show 25–31 items because of the variance in the number of "nested sets" of items on each test form. As shown in Figure 1, a single item may cross four sequence numbers.

students undertook the tests. A total of 577 students (301 from grade 4 and 276 from grade 5) were each administered one of the four test forms. The following information provides some specifics of the first moderate-scale field test:

- All four test forms were spiraled in order to achieve a random distribution of test forms to both grade 4 and grade 5 students.
- A key entry vendor was utilized to process the raw results from the answer documents.
- A maximum of 60 minutes was given to complete the test. Most students finished within a 25–35 minute time period.
- Data analysts and psychometricians provided classical statistics for item analysis.
- Following analysis and review of each item, some minor edits were made to a few items in order to improve wording and avoid confusion.

The statistics collected from the Spring 2010 administration enabled developers to identify a set of linking items (12) that would be included on each of the Fall 2010 test forms. A decision was also made at this time to reduce the test length from 50 to 46 items. The next phase of the project was to conduct large-scale testing of more than 5,000 students (2,616 from grade 4 and 2,527 from grade 5) in five urban districts on the East Coast of the U.S. In preparation for the Fall 2010 administration, a short practice test was distributed for use two weeks prior to the scheduled testing dates. This practice test provided students an opportunity to become familiar with the test questions and answer formats at a time when their teacher could offer support and guidance. Additionally, a test administration manual was created, and test and site administrators were given training on how to administer the tests.

While item development continued throughout the Fall of 2010 and Winter of 2010/2011, four operational test forms were administered as pre- and post-tests in the five districts described above. Decisions about which items would be administered to grade 4 students versus grade 5 students were made based largely upon the field-test results from the Spring of 2010 as well as content alignment to the new CCSSM.

Forms A and B were administered to grade 4 students, and Forms C and D were administered to grade 5 students. As stated above, more than 5,000 students and 248 teachers participated in the Fall 2010 pre-test. However, participation in the Spring 2011 post-test administration was even greater, increasing to almost 7,000 students (3,359 from grade 4 and 3,380 from grade 5) and 328 teachers. Although statistical analyses were completed during both test administrations, it was known that the final test forms would be revised to include Geometry/Measurement items (which had not yet been piloted). Some observations concerning the administrations of these early forms include:

- All item P+ values increased from the Fall 2010 administration to the Spring 2011 administration.
- For each form, 60% of the items represented the Number/Operations domain, and 40% of the items represented the Algebra domain.
- Twelve items were linking (common to all forms) in order to create a vertical scale.

- While all forms showed signs of a speededness issue[7], the Spring rates of omission at the end of the test were much reduced.
- In the Fall administration, the test forms were alternated throughout the classroom. Students who were administered Form A in the pre-test were assigned Form B in the post-test (and vice versa). The same controls were put in place for Forms C and D.
- Scale scores were assigned to students in order to show growth from the pre-test to the post-test.
- Students showed some difficulty in responding to the gridded-response items (high omit rates), though this was less of an issue in the post-test administration.
    - Gridded-response items presented a unique experience for most students.
    - Greater emphasis was placed on making sure that teachers provided students with the practice test before the Spring administration so students would know how to properly respond to such item types.

Concurrent with the Spring 2011 operational administration, pilot testing of six new test modules (Forms P, Q, S, V, W, and X) was conducted. These new modules included geometry items; the last of the three advisory panels was convened in June of 2010, with panel members suggesting prototype items and insights from their own work.[8] Item development began following this meeting, with the advantage that by that time, drafts of the CCSSM were available and could play a role in the review process. By spring of 2011, these items were ready for pilot in Florida schools. Co-PI Judy Hickman was again the test administrator. The modules were randomly assigned and administered to 252 grade 4 students and 196 grade 5 students in May 2011. Each module contained a mix of 46 Geometry/Measurement and Number/Operations items. Informal interviews were conducted with more than 30 students immediately following the administration of the pilot-test modules. The main takeaway from the interviews was that some Geometry/Measurement concepts were not yet being taught in the schools. Since the CCSSM were still relatively new at the time, it was not a surprising finding.

Statistics were compiled for the new Geometry/Measurement and Number/Operations items in preparation for the compilation of the final eight test modules. While initially it was planned for test modules to be constructed to measure each of the three domains separately, it was determined that a better test design would be to keep all three domains together in the final test modules. While the developers agreed that there is a potential use for testing a student's knowledge in each of these domains separately, it was decided to keep all three domains together and report a single score per student. A factor in this decision was the possibility that while some students would be administered a test containing more familiar Number/Operations topics, a student who is administered a Geometry/Measurement test might be more prone to giving up when faced with many items that test unfamiliar or challenging concepts.

Since individual item statistics are contained in the tables in the next section of this report, researchers could of their own accord create mini-testlets in order to assess understanding in these domains separately. Of course, such a change in the format of any test would need to come

[7] A test is considered to exhibit "speededness" issues if there is evidence (omit rates greater than 5%) that students are running out of time toward the end of the test.
[8] Michael Battista, Jack Smith, Grayson Wheatley

with the usual disclaimer that statistics may not hold true for such a dramatic change in test design.

The next step was to create a test blueprint for the final eight test modules. Because much of the elementary mathematics curriculum is focused on the Number/Operations domain, a higher percentage of test items was allocated to measure this content area than either the Algebra or Geometry/Measurement domains. While slight variances occurred across the eight forms (due to differences in the number of items and the available points of the "nested" sets), the approximate percentage of items on each test form, by domain, is: Number/Operations 50%, Algebra 25%, and Geometry/Measurement 25%. The linking set was also modified to reflect similar percentages.

Relying upon the statistics from the 2010/2011 test forms, items were again evaluated to see whether they were good candidates for inclusion on the final form. Some minor edits were made to a few items—again, to make the items clearer for students. The result was eight test modules: four modules (Forms F, G, H, and J) to be administered to grade 4 students, and four modules (Forms K, L, and M, N) to be administered to grade 5 students. These forms contained between 26 and 29 items (counting "nested sets" as one item). These eight test modules were administered in Fall 2011 to 3,265 grade 4 students and 2,963 grade 5 students, and then in Spring 2012 to 3,080 grade 4 students and 2,840 grade 5 students (see below for more details on the sample). As noted by the n-counts, some attrition occurred between the pre- and post-test administrations. However, 84% of the classroom teachers enrolled in either the NCTE or Math Solutions study since Fall 2010 were still part of the study in Spring 2012. Observations related to these two operational administrations include:

- As many grade 5 students as possible who had participated in the original administrations in Fall 2010/Spring 2011 were included in the Fall 2011/Spring 2012 administrations in order to evaluate the effectiveness of the vertical scale.
- Spiraling of the test forms occurred during the administration of the pre-test, and care was taken to make sure students did not receive the same form in the post-test administration.
- The reliabilities of the test forms range from a low of 0.82 to a high of 0.89.
- Speededness was still an issue for all test forms, though lower rates of omits were observed than when the original test modules were administered in Fall 2010/Spring 2011.

In summary, the result is that there are eight final test forms that are believed to be good instruments for measuring student growth across the domains of Number/Operations, Algebra, and Geometry/Measurement. These eight forms have been prepared for use and have answer documents and keys. They are included in the final PDF packet of materials available from the Harvard Graduate School of Education. A test administration manual and practice test are also included in the packet of test materials. All of the materials can be reprinted for use for educational purposes only, not for commercial ventures.

Sample details

The eight test forms were administered in 2011-2012 to students involved in the National Center for Teacher Effectiveness field study. This study included four urban districts. Table S-1 shows the characteristics of the sample for the spring 2012 test.

Table S-1: Student and school totals by district

SEMESTER 2

District	Grade 4	Grade 5	Total
11	549	398	947
12	563	496	1059
13	205	180	385
14	947	1022	1969
Total	2264	2096	4360

SCHOOLS

District	Number
11	14
12	9
13	8
14	12
Total	43

Table S-2: Student characteristics by district

District/Demo	N	Percent
African American	4798	42
11	1045	39
12	1118	52
13	425	62
14	2210	34
Asian	4798	7
11	1045	12
12	1118	3
13	425	3
14	2210	8
Hispanic	4798	22
11	1045	37
12	1118	14
13	425	11
14	2210	22
White	4798	24
11	1045	6
12	1118	26
13	425	22
14	2210	31
Other	4798	4
11	1045	5
12	1118	5
13	425	3

District/Demo	N	Percent
Male	4798	51
11	1045	48
12	1118	53
13	425	50
14	2210	51
Free/Reduced Price Lunch	4777	64
11	1045	78
12	1097	75
13	425	54
14	2210	53
Special Ed	4777	13
11	1045	18
12	1097	13
13	425	10
14	2210	11
Limited English Proficiency	4777	19
11	1045	36
12	1097	24
13	425	4
14	2210	12

	14	2210	4

Psychometric Analyses

Our psychometric analyses had three major goals:

1. Determine the characteristics of test items and forms
2. Equate forms to one another
3. Ensure vertical equating between test forms covering grades 4 and 5.

For the statistical analyses reported below, only the sample from the Spring 2012 administration was used because items performed better in the Spring 2012 administration than in the Fall 2011 administration. We attribute the better item performance to the fact that test forms were designed to cover material taught in each grade; using these test forms prior to students learning the material likely increased measurement error. The following categories of students were excluded from the calibration analyses: (a) those taking the same test forms in the Fall and Spring administrations, and (b) those attempting fewer than 10 items. In all analyses, omitted responses were treated as 0s, and not-reached responses were treated as missing.

Classical Item Statistics

To determine the characteristics of test items and forms, we conducted both classical test theory analyses and Item Response Theory analyses. Tables 1–8 describe these classical test theory statistics, including the sample size (*N*), mean, standard deviation, P+ value (mean/max possible item score), omit rate, and not-reached rate for each item in the eight test forms, respectively. All item statistics were within reasonable ranges except that all test forms appeared to have a speededness issue, as the not-reached rates were quite high toward the end of the test for all forms.

Table 1.
Grade 4 Form F: Item Statistics

Item ID	*N*	Mean	*SD*	P+	Polyserial item-total correlation	Omit rate (%)	Not reached rate (%)
HVD10130	716	.51	.50	.51	.59	.14	.00
HVD10017	716	1.29	.79	.65	.45	.70	.00
HVD90101	716	.78	.42	.78	.42	.42	.00
HVD10129	716	.69	.46	.69	.67	.00	.00
HVD90229	716	.15	.36	.15	.55	3.77	.00
HVD30055	716	.46	.50	.46	.74	3.35	.00
HVD10125	716	.72	.45	.72	.70	2.37	.14
HVD10325	716	.76	.43	.76	.70	.70	.28
HVD30066	716	.36	.48	.36	.24	1.40	.28
HVD30079	716	.69	.46	.69	.59	1.96	.28
HVD30056	716	.57	.50	.57	.46	1.12	.28
HVD30019	716	.66	.47	.66	.49	2.37	.56
HVD30085	716	.53	.50	.53	.59	1.68	.56
HVD10134	716	.94	.80	.47	.48	.84	1.12
HVD30050	716	.49	.77	.25	.54	2.23	1.54
HVD10021	716	.77	.81	.26	.69	1.26	2.79
HVD10119	716	.82	.39	.82	.67	.70	4.19
HVD10033	716	.60	.49	.60	.64	1.82	4.47
HVD30024	716	.80	.86	.40	.66	0.84	5.59
HVD90110	716	.58	.49	.58	.58	1.12	7.26
HVD10032	716	.42	.49	.42	.43	1.82	8.66
HVD90042	716	.38	.49	.38	.48	.42	9.64
HVD10123	716	.69	.46	.69	.64	2.23	10.89
HVD10091	716	.54	.50	.54	.61	.84	12.29
HVD30046	716	.37	.48	.37	.40	2.65	13.69
HVD30246	716	.39	.49	.39	.62	.28	14.53
HVD30346	716	.13	.33	.13	.68	.14	16.48

Note. P+ = mean/max possible item score.

Table 2.
Grade 4 Form G: Item Statistics

Item ID	*N*	Mean	*SD*	P+	Polyserial item-total correlation	Omit rate (%)	Not reached rate (%)
HVD10289	780	.77	.42	.77	.64	1.67	.00
HVD10017	780	1.23	.82	.62	.47	.38	.00
HVD90101	780	.76	.43	.76	.35	.51	.00
HVD10204	780	.78	.79	.39	.50	1.03	.00
HVD30004	780	.37	.48	.37	.64	2.56	.00
HVD10125	780	.67	.47	.67	.70	3.46	.00
HVD10325	780	.75	.43	.75	.71	.90	.00
HVD30066	780	.37	.48	.37	.33	2.18	.13
HVD10001	780	.77	.42	.77	.55	.51	.26
HVD30040	780	.50	.50	.50	.43	1.92	.64
HVD30081	780	.24	.43	.24	.20	2.18	1.03
HVD30085	780	.57	.50	.57	.60	1.28	1.03
HVD10134	780	.94	.79	.47	.51	1.28	1.15
HVD10019	780	1.10	.53	.55	.53	.13	1.28
HVD10107	780	.64	.48	.64	.73	3.21	1.67
HVD10307	780	.38	.49	.38	.70	3.33	2.31
HVD10407	780	.22	.42	.22	.58	3.85	2.82
HVD10507	780	.50	.50	.50	.44	3.08	2.95
HVD10119	780	.82	.39	.82	.62	1.28	3.33
HVD10033	780	.57	.49	.57	.64	1.92	3.97
HVD30018	780	.93	.77	.47	.53	.64	4.36
HVD30063	780	.18	.38	.18	.49	2.18	5.00
HVD10032	780	.38	.49	.38	.44	1.41	5.90
HVD90042	780	.36	.48	.36	.45	.51	7.44
HVD30036	780	.24	.43	.24	.46	2.31	8.85
HVD10091	780	.50	.50	.50	.57	.51	9.62
HVD30046	780	.34	.47	.34	.47	3.08	11.54
HVD30246	780	.45	.50	.45	.53	.64	13.33
HVD30346	780	.10	.30	.10	.63	.51	14.10

Note. P+ = mean/max possible item score.

Table 3.
Grade 4 Form H: Item Statistics

Item ID	*N*	Mean	*SD*	P+	Polyserial item-total correlation	Omit rate (%)	Not reached rate (%)
HVD30082	771	.52	.50	.52	.42	.91	.00
HVD10017	771	1.23	.80	.62	.54	.52	.00
HVD90101	771	.78	.41	.78	.39	.65	.00
HVD10047	771	1.32	.77	.66	.59	1.82	.00
HVD90229	771	.16	.37	.16	.60	5.58	.00
HVD30065	771	.46	.50	.46	.68	3.63	.00
HVD10125	771	.70	.46	.70	.73	2.98	.26
HVD10325	771	.77	.42	.77	.79	.78	.26
HVD30066	771	.38	.49	.38	.32	1.95	.26
HVD10040	771	1.24	.73	.41	.62	.00	.26
HVD30008	771	.66	.47	.66	.31	1.69	.52
HVD10134	771	1.02	.79	.51	.53	1.17	.52
HVD30042	771	.93	.88	.47	.72	1.17	.78
HVD30005	771	.14	.34	.14	.63	4.41	1.17
HVD10026	771	.39	.49	.39	.56	4.28	1.56
HVD30057	771	.12	.33	.12	.47	4.28	2.08
HVD30078	771	.23	.42	.23	.58	.91	2.33
HVD10119	771	.82	.38	.82	.62	1.69	2.46
HVD10033	771	.59	.49	.59	.68	1.17	3.11
HVD10102	771	.60	.63	.30	.35	.39	4.28
HVD30077	771	.53	.50	.53	.16	.39	5.58
HVD10032	771	.42	.49	.42	.37	1.56	7.65
HVD90042	771	.33	.47	.33	.38	.91	8.82
HVD10123	771	.73	.44	.73	.66	1.95	10.25
HVD10091	771	.50	.50	.50	.47	.39	10.89
HVD30046	771	.39	.49	.39	.54	2.08	12.19
HVD30246	771	.40	.49	.40	.59	.00	14.40
HVD30346	771	.12	.32	.12	.66	.00	15.69

Note. P+ = mean/max possible item score.

Table 4.
Grade 4 Form J: Item Statistics

Item ID	*N*	Mean	*SD*	P+	Polyserial item-total correlation	Omit rate (%)	Not reached rate (%)
HVD30083	785	.51	.50	.51	.53	1.02	.00
HVD10017	785	1.26	.82	.63	.58	.00	.00
HVD90101	785	.76	.43	.76	.49	.64	.00
HVD10100	785	1.35	.55	.68	.44	.51	.00
HVD30028	785	.59	.49	.59	.48	.76	.00
HVD30075	785	.69	.46	.69	.55	1.27	.00
HVD10125	785	.68	.47	.68	.70	3.31	.13
HVD10325	785	.76	.43	.76	.70	.64	.13
HVD30066	785	.35	.48	.35	.29	1.78	.13
HVD30039	785	.87	.84	.44	.66	3.06	.13
HVD10134	785	1.01	.80	.51	.58	1.15	.25
HVD30001	785	.51	.62	.26	.35	1.53	.76
HVD30005	785	.12	.33	.12	.75	5.86	2.04
HVD10014	785	.73	.45	.73	.52	1.78	2.29
HVD10314	785	.50	.50	.50	.69	3.31	2.80
HVD10414	785	.45	.50	.45	.53	3.57	3.57
HVD10119	785	.80	.40	.80	.63	.89	3.82
HVD10033	785	.58	.49	.58	.63	1.02	4.33
HVD10102	785	.51	.63	.26	.32	0.38	4.59
HVD30077	785	.46	.50	.46	.07	.76	5.35
HVD10032	785	.43	.50	.43	.49	1.53	6.24
HVD90042	785	.38	.49	.38	.43	.76	7.39
HVD10123	785	.70	.46	.70	.66	1.91	8.15
HVD10091	785	.50	.50	.50	.52	.76	8.79
HVD30046	785	.36	.48	.36	.50	3.06	11.21
HVD30246	785	.44	.50	.44	.58	.00	12.48
HVD30346	785	.13	.34	.13	.74	.00	14.39

Note. P+ = mean/max possible item score.

Table 5.
Grade 5 Form K: Item Statistics

Item ID	*N*	Mean	*SD*	P+	Polyserial item-total correlation	Omit rate (%)	Not reached rate (%)
HVD10131	700	.67	.47	.67	.70	.29	.00
HVD30035	700	.31	.46	.31	.54	.57	.00
HVD10037	700	.78	.79	.39	.63	.57	.00
HVD30095	700	.44	.50	.44	.52	.57	.00
HVD30027	700	.55	.50	.55	.59	1.43	.00
HVD30006	700	.46	.50	.46	.54	.71	.00
HVD30306	700	.10	.31	.10	.53	1.57	.00
HVD10125	700	.81	.39	.81	.62	2.00	.00
HVD10325	700	.86	.35	.86	.69	.00	.00
HVD30066	700	.53	.50	.53	.50	.43	.00
HVD30090	700	.47	.50	.47	.58	.43	.14
HVD30069	700	.57	.50	.57	.62	1.57	.29
HVD10203	700	.54	.50	.54	.46	.57	.29
HVD30031	700	.69	.46	.69	.61	1.00	.29
HVD10134	700	1.22	.82	.61	.53	.00	.57
HVD10043	700	.98	.77	.49	.67	.29	.86
HVD30255	700	.66	.47	.66	.70	.57	2.29
HVD30020	700	.21	.40	.21	.60	1.71	2.71
HVD30091	700	.45	.50	.45	.46	2.00	3.14
HVD10119	700	.84	.37	.84	.64	.57	4.00
HVD10033	700	.71	.46	.71	.72	1.14	4.43
HVD10011	700	1.02	.56	.51	.37	.43	5.00
HVD30043	700	.83	.89	.42	.63	.29	5.43
HVD30194	700	.67	.47	.67	.56	1.00	7.14
HVD10106	700	.50	.50	.50	.57	.29	7.57
HVD30046	700	.53	.50	.53	.64	1.86	8.29
HVD30246	700	.55	.50	.55	.52	.71	9.14
HVD30346	700	.34	.47	.34	.73	.43	9.43
HVD10026	700	.56	.50	.56	.63	.00	11.86

Note. P+ = mean/max possible item score.

Table 6.
Grade 5 Form L: Item Statistics

Item ID	*N*	Mean	*SD*	P+	Polyserial item-total correlation	Omit rate (%)	Not reached rate (%)
HVD90204	728	.73	.44	.73	.51	.27	.00
HVD30033	728	.37	.48	.37	.56	.00	.00
HVD30013	728	.95	.86	.48	.59	.55	.00
HVD10025	728	.51	.50	.51	.69	.00	.00
HVD30052	728	.35	.48	.35	.66	.82	.00
HVD10122	728	.72	.45	.72	.61	1.24	.00
HVD10322	728	.39	.49	.39	.57	2.47	.00
HVD10125	728	.80	.40	.80	.71	2.34	.14
HVD10325	728	.86	.34	.86	.72	.14	.14
HVD30066	728	.54	.50	.54	.50	.41	.27
HVD30049	728	.70	.46	.70	.58	.69	.41
HVD30041	728	.44	.50	.44	.72	.41	.55
HVD30030	728	.45	.50	.45	.49	.41	.69
HVD30080	728	.30	.46	.30	.42	1.24	.82
HVD10134	728	1.20	.80	.60	.56	.27	.82
HVD10023	728	2.02	.92	.67	.74	.00	.82
HVD10345	728	.49	.50	.49	.65	1.79	.96
HVD30022	728	.32	.47	.32	.52	1.37	1.65
HVD10118	728	.74	.44	.74	.73	1.37	2.34
HVD10119	728	.88	.32	.88	.65	.27	2.75
HVD10033	728	.66	.47	.66	.67	1.37	3.02
HVD10011	728	1.04	.57	.52	.39	.14	3.30
HVD30043	728	.89	.92	.45	.73	.41	3.98
HVD30063	728	.47	.50	.47	.51	.69	4.40
HVD10108	728	.37	.48	.37	.56	.14	4.95
HVD30046	728	.51	.50	.51	.58	1.92	5.49
HVD30246	728	.53	.50	.53	.57	.41	5.91
HVD30346	728	.28	.45	.28	.72	.55	6.46
HVD30017	728	.28	.45	.28	.62	.00	7.28

Note. P+ = mean/max possible item score.

Table 7.
Grade 5 Form M: Item Statistics

Item ID	*N*	Mean	*SD*	P+	Polyserial item-total correlation	Omit rate (%)	Not reached rate (%)
HVD10111	704	.63	.48	.63	.68	.00	.00
HVD30069	704	.51	.50	.51	.61	1.42	.00
HVD10132	704	1.36	.65	.45	.46	.14	.00
HVD30093	704	.32	.47	.32	.51	.28	.00
HVD10124	704	.54	.50	.54	.39	.99	.00
HVD10122	704	.77	.42	.77	.71	1.28	.00
HVD10322	704	.43	.50	.43	.68	2.41	.00
HVD10125	704	.80	.40	.80	.67	1.85	.00
HVD10325	704	.87	.33	.87	.70	.28	.00
HVD30066	704	.54	.50	.54	.47	.85	.00
HVD10029	704	1.20	.85	.60	.66	.71	.00
HVD10134	704	1.20	.83	.60	.63	.43	.43
HVD10043	704	.92	.75	.46	.66	.14	.57
HVD30060	704	.39	.49	.39	.67	1.42	1.28
HVD30016	704	.34	.47	.34	.49	1.28	1.42
HVD30174	704	.12	.33	.12	.60	1.70	1.56
HVD10119	704	.86	.35	.86	.53	.43	1.70
HVD10033	704	.71	.46	.71	.71	.99	1.85
HVD10011	704	1.03	.56	.52	.31	.14	1.85
HVD30047	704	.98	.58	.49	.54	.43	2.70
HVD30079	704	.77	.42	.77	.61	.85	3.69
HVD10106	704	.51	.50	.51	.46	.43	4.55
HVD30046	704	.52	.50	.52	.61	1.14	5.54
HVD30246	704	.54	.50	.54	.54	.43	5.54
HVD30346	704	.28	.45	.28	.67	.43	5.54
HVD90942	704	.36	.48	.36	.55	.00	7.24

Note. P+ = mean/max possible item score.

Table 8.
Grade 5 Form N: Item Statistics

Item ID	*N*	Mean	*SD*	P+	Polyserial item-total correlation	Omit rate (%)	Not reached rate (%)
HVD10012	691	.85	.35	.85	.57	.43	.00
HVD30074	691	.73	.44	.73	.55	.43	.00
HVD30032	691	1.06	.87	.53	.68	.29	.00
HVD30048	691	.73	.45	.73	.54	.58	.00
HVD30071	691	.47	.50	.47	.70	1.16	.00
HVD30006	691	.47	.50	.47	.58	.58	.00
HVD30306	691	.12	.33	.12	.66	1.59	.00
HVD10125	691	.79	.41	.79	.70	2.17	.00
HVD10325	691	.85	.36	.85	.71	.14	.00
HVD30066	691	.48	.50	.48	.52	.43	.00
HVD30025	691	.39	.49	.39	.58	.29	.00
HVD30088	691	.38	.48	.38	.51	.14	.00
HVD10203	691	.52	.50	.52	.40	.58	.00
HVD10024	691	.42	.49	.42	.38	1.45	.00
HVD10134	691	1.19	.81	.60	.56	.00	.14
HVD10023	691	2.01	.94	.67	.74	.29	.29
HVD10045	691	.34	.47	.34	.73	3.47	.72
HVD10086	691	.45	.50	.45	.68	4.78	1.16
HVD30062	691	.55	.50	.55	.47	.72	2.17
HVD10119	691	.85	.35	.85	.68	.72	2.32
HVD10033	691	.67	.47	.67	.71	.14	2.46
HVD10011	691	1.03	.57	.52	.43	.00	2.46
HVD30043	691	.76	.88	.38	.62	.43	2.75
HVD30090	691	.43	.50	.43	.56	1.01	3.62
HVD10108	691	.47	.50	.47	.55	.29	5.21
HVD30046	691	.53	.50	.53	.67	2.03	6.51
HVD30246	691	.54	.50	.54	.62	.43	6.80
HVD30346	691	.38	.49	.38	.74	.43	7.53
HVD30055	691	.50	.50	.50	.76	.00	9.55

Note. P+ = mean/max possible item score.

Item Calibration and Equating by IRT Model

The four grade 4 forms and the four grade 5 forms were equated to a common scale using common items and GPCM. In particular, the four grade 4 tests were calibrated concurrently using GPCM with common items between test forms as anchor items, and the same was done for the four grade 5 tests. Then, the grades 4 and 5 test forms were linked together based on the nine linking items across all test forms using the Stocking and Lord procedure (Stocking & Lord, 1983). IRT calibration and equating were conducted in GENASYS, proprietary statistical analysis software developed by Educational Testing Service (ETS).

In this report, the GPCM is formulized as follows:

$$P_{ijm} = p(x_{ij} = m \mid \theta_j, a_i, b_i, \mathbf{d}_i) = \frac{\exp\left[\sum_{h=0}^{m} 1.7a_i(\theta_j - b_i + d_{ih})\right]}{\sum_{v=0}^{M_i-1} \exp\left[\sum_{h=0}^{v} 1.7a_i(\theta_j - b_i + d_{ih})\right]},$$

where

$a_i(\theta_j - b_i + d_{i0}) \equiv 0$;

x_{ij} is examinee j's score on item i;

M_i is item i's number of score categories;

m is item i's possible integer score point equal to 0 to $M_i - 1$;

a_i is the discrimination (slope) parameter for item i;

b_i is the difficulty (local) parameter for item *i*;

d_{ih} is the step parameter[9] for score h of item *i*, and for a dichotomous item having only two score categories (0 and 1) $d_{i1} \equiv 0$;

$\mathbf{d}_i$ is the vector with elements d_{ih};

θ_j is examinee j's ability (theta) score; and

P_{ijm} is examinee j's probability of achieving score m on item i.

Tables 9–16 list the item parameter estimates after equating for the eight test forms, respectively. All item parameter estimates as well as item fit were reasonable.

[9] The step parameter can be interpreted as the relative difficulty moving from a low score category to a high score category. For example, for Item HVD10017 in Table 9 the step parameter for score 1 is -.07, and for score 2 is .07. So, it is easier for students to reach score 1 from score 0 than to reach score 2 from score 1.

Table 9.
Grade 4 Form F: GPCM Item Parameter Estimates

Item ID	Discrimination (a_i)		Difficulty (b_i)		Step parameter for score 1 (d_{i1})		Step parameter for score 2 (d_{i2})		Step parameter for score 3 (d_{i3})	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
HVD10130	.69	.07	-.07	.07						
HVD10017	.31	.02	-.82	.06	-.07	.10	.07	.08		
HVD90101	.39	.03	-1.98	.15						
HVD10129	.87	.09	-.77	.08						
HVD90229	.72	.06	1.74	.11						
HVD30055	1.19	.10	.10	.05						
HVD10125	1.11	.05	-.70	.03						
HVD10325	1.18	.06	-.93	.03						
HVD30066	.23	.02	1.45	.17						
HVD30079	.67	.08	-.90	.10						
HVD30056	.46	.06	-.38	.11						
HVD30019	.49	.06	-.89	.13						
HVD30085	.72	.05	-.21	.05						
HVD10134	.36	.02	.09	.04	.48	.07	-.48	.08		
HVD30050	.37	.03	1.38	.12	-1.01	.17	1.01	.21		
HVD10021	.74	.05	1.17	.05	1.35	.07	-.61	.11	-.74	.18
HVD10119	.92	.05	-1.19	.05						
HVD10033	.89	.04	-.25	.03						
HVD30024	.56	.04	.49	.06	-.25	.11	.25	.12		
HVD90110	.73	.07	-.15	.07						
HVD10032	.46	.03	.68	.06						
HVD90042	.47	.03	.98	.07						
HVD10123	.86	.05	-.55	.04						
HVD10091	.63	.03	.20	.04						
HVD30046	.56	.03	.93	.06						
HVD30246	.75	.04	.56	.04						
HVD30346	1.10	.07	1.76	.06						

Table 10.
Grade 4 Form G: GPCM Item Parameter Estimates

Item ID	Discrimination		Difficulty		Step parameter for score 1 (d_{i1})		Step parameter for score 2 (d_{i2})	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
HVD10289	.83	.09	-1.16	.10				
HVD10017	.31	.02	-.82	.06	-.07	.10	.07	.08
HVD90101	.39	.03	-1.98	.15				
HVD10204	.32	.03	.71	.11	.29	.16	-.29	.19
HVD30004	.82	.08	.48	.07				
HVD10125	1.11	.05	-.70	.03				
HVD10325	1.18	.06	-.93	.03				
HVD30066	.23	.02	1.45	.17				
HVD10001	.62	.07	-1.38	.14				
HVD30040	.43	.05	.00	.10				
HVD30081	.22	.04	3.20	.65				
HVD30085	.72	.05	-.21	.05				
HVD10134	.36	.02	.09	.04	.48	.07	-.48	.08
HVD10019	.61	.03	-.39	.07	1.93	.12	-1.93	.10
HVD10107	1.16	.11	-.47	.05				
HVD10307	1.10	.10	.38	.05				
HVD10407	.82	.08	1.19	.11				
HVD10507	.48	.05	.04	.09				
HVD10119	.92	.05	-1.19	.05				
HVD10033	.89	.04	-.25	.03				
HVD30018	.40	.04	.29	.08	.63	.12	-.63	.14
HVD30063	.57	.07	1.92	.21				
HVD10032	.46	.03	.68	.06				
HVD90042	.47	.03	.98	.07				
HVD30036	.52	.06	1.62	.19				
HVD10091	.63	.03	.20	.04				
HVD30046	.56	.03	.93	.06				
HVD30246	.75	.04	.56	.04				
HVD30346	1.10	.07	1.76	.06				

Table 11.
Grade 4 Form H: GPCM Item Parameter Estimates

Item ID	Discrimination (a_i)		Difficulty (b_i)		Step parameter for score 1 (d_{i1})		Step parameter for score 2 (d_{i2})		Step parameter for score 3 (d_{i3})	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
HVD30082	.39	.05	-.10	.12						
HVD10017	.31	.02	-.82	.06	-.07	.10	.07	.08		
HVD90101	.39	.03	-1.98	.15						
HVD10047	.46	.04	-.82	.08	.34	.14	-.34	.11		
HVD90229	.72	.06	1.74	.11						
HVD30065	.92	.08	.18	.06						
HVD10125	1.11	.05	-.70	.03						
HVD10325	1.18	.06	-.93	.03						
HVD30066	.23	.02	1.45	.17						
HVD10040	.48	.02	-.18	.07	3.61	.20	-4.30	.25	.69	.28
HVD30008	.29	.05	-1.39	.26						
HVD10134	.36	.02	.09	.04	.48	.07	-.48	.08		
HVD30042	.59	.04	.17	.05	-.28	.10	.28	.10		
HVD30005	1.08	.08	1.60	.07						
HVD10026	.64	.07	.55	.09						
HVD30057	.57	.08	2.41	.27						
HVD30078	.75	.07	1.28	.11						
HVD10119	.92	.05	-1.19	.05						
HVD10033	.89	.04	-.25	.03						
HVD10102	.25	.02	2.55	.15	1.71	.13	-1.71	.24		
HVD30077	.13	.03	.60	.25						
HVD10032	.46	.03	.68	.06						
HVD90042	.47	.03	.98	.07						
HVD10123	.86	.05	-.55	.04						
HVD10091	.63	.03	.20	.04						
HVD30046	.56	.03	.93	.06						
HVD30246	.75	.04	.56	.04						
HVD30346	1.10	.07	1.76	.06						

Table 12.
Grade 4 Form J: GPCM Item Parameter Estimates

Item ID	Discrimination (a_i)		Difficulty (b_i)		Step parameter for score 1 (d_{i1})		Step parameter for score 2 (d_{i2})	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
HVD30083	.56	.05	-.03	.08				
HVD10017	.31	.02	-.82	.06	-.07	.10	.07	.08
HVD90101	.39	.03	-1.98	.15				
HVD10100	.38	.02	-2.01	.11	2.76	.30	-2.76	.12
HVD30028	.49	.05	-.49	.10				
HVD30075	.60	.06	-.93	.11				
HVD10125	1.11	.05	-.70	.03				
HVD10325	1.18	.06	-.93	.03				
HVD30066	.23	.02	1.45	.17				
HVD30039	.52	.04	.30	.06	.11	.10	-.11	.11
HVD10134	.36	.02	.09	.04	.48	.07	-.48	.08
HVD30001	.24	.03	2.82	.24	1.75	.19	-1.75	.37
HVD30005	1.08	.08	1.60	.07				
HVD10014	.63	.07	-1.03	.11				
HVD10314	.94	.08	.02	.06				
HVD10414	.56	.06	.33	.09				
HVD10119	.92	.05	-1.19	.05				
HVD10033	.89	.04	-.25	.03				
HVD10102	.25	.02	2.55	.15	1.71	.13	-1.71	.24
HVD30077	.13	.03	.60	.25				
HVD10032	.46	.03	.68	.06				
HVD90042	.47	.03	.98	.07				
HVD10123	.86	.05	-.55	.04				
HVD10091	.63	.03	.20	.04				
HVD30046	.56	.03	.93	.06				
HVD30246	.75	.04	.56	.04				
HVD30346	1.10	.07	1.76	.06				

Table 13.
Grade 5 Form K: GPCM Item Parameter Estimates

Item ID	Discrimination (a_i)		Difficulty (b_i)		Step parameter for score 1 (d_{i1})		Step parameter for score 2 (d_{i2})	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
HVD10131	.80	.07	-.07	.08				
HVD30035	.48	.05	1.95	.15				
HVD10037	.42	.03	1.41	.08	.50	.13	-.50	.15
HVD30095	.43	.05	1.12	.12				
HVD30027	.54	.05	.45	.10				
HVD30006	.48	.04	.94	.07				
HVD30306	.57	.05	3.39	.18				
HVD10125	.83	.04	-.76	.05				
HVD10325	.92	.05	-1.10	.06				
HVD30066	.40	.02	.55	.06				
HVD30090	.52	.04	1.04	.07				
HVD30069	.58	.04	.51	.06				
HVD10203	.32	.03	.50	.11				
HVD30031	.60	.06	-.30	.11				
HVD10134	.33	.01	.05	.05	.08	.09	-.08	.08
HVD10043	.52	.03	.89	.05	.88	.08	-.88	.08
HVD30255	.84	.07	.06	.08				
HVD30020	.59	.07	2.52	.16				
HVD30091	.39	.04	1.21	.13				
HVD10119	.73	.04	-1.11	.07				
HVD10033	.85	.04	-.05	.04				
HVD10011	.29	.01	.73	.07	3.13	.11	-3.13	.11
HVD30043	.41	.01	1.23	.04	-.89	.09	.89	.10
HVD30194	.59	.05	.07	.10				
HVD10106	.49	.04	.86	.07				
HVD30046	.68	.03	.76	.04				
HVD30246	.58	.03	.69	.04				
HVD30346	.89	.04	1.64	.04				
HVD10026	.71	.06	.74	.08				

Table 14.
Grade 5 Form L: GPCM Item Parameter Estimates

Item ID	Discrimination (a_i)		Difficulty (b_i)		Step parameter for score 1 (d_{i1})		Step parameter for score 2 (d_{i2})		Step parameter for score 3 (d_{i3})	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
HVD90204	.46	.05	-.86	.17						
HVD30033	.53	.05	1.41	.11						
HVD30013	.34	.03	.82	.09	-.24	.17	.24	.17		
HVD10025	.77	.07	.63	.07						
HVD30052	.69	.06	1.43	.09						
HVD10122	.73	.05	-.52	.07						
HVD10322	.62	.04	1.15	.06						
HVD10125	.83	.04	-.76	.05						
HVD10325	.92	.05	-1.10	.06						
HVD30066	.40	.02	.55	.06						
HVD30049	.55	.05	-.47	.12						
HVD30041	.87	.07	.92	.07						
HVD30030	.40	.04	1.05	.12						
HVD30080	.36	.04	2.28	.22						
HVD10134	.33	.01	.05	.05	.08	.09	-.08	.08		
HVD10023	.59	.03	-.38	.04	1.73	.13	-.52	.08	-1.21	.07
HVD10345	.68	.06	.77	.08						
HVD30022	.46	.05	1.90	.15						
HVD10118	.99	.09	-.30	.08						
HVD10119	.73	.04	-1.11	.07						
HVD10033	.85	.04	-.05	.04						
HVD10011	.29	.01	.73	.07	3.13	.11	-3.13	.11		
HVD30043	.41	.01	1.23	.04	-.89	.09	.89	.10		
HVD30063	.47	.05	.97	.11						
HVD10108	.52	.04	1.28	.08						
HVD30046	.68	.03	.76	.04						
HVD30246	.58	.03	.69	.04						
HVD30346	.89	.04	1.64	.04						
HVD30017	.62	.07	1.99	.12						

Table 15.
Grade 5 Form M: GPCM Item Parameter Estimates

Item ID	Discrimination (a_i)		Difficulty (b_i)		Step parameter for score 1 (d_{i1})		Step parameter for score 2 (d_{i2})		Step parameter for score 3 (d_{i3})	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
HVD10111	.75	.07	.12	.08						
HVD30069	.58	.04	.51	.06						
HVD10132	.28	.02	.85	.13	5.83	.37	-1.34	.18	-4.49	.39
HVD30093	.44	.05	1.90	.16						
HVD10124	.28	.04	.38	.17						
HVD10122	.73	.05	-.52	.07						
HVD10322	.62	.04	1.15	.06						
HVD10125	.83	.04	-.76	.05						
HVD10325	.92	.05	-1.10	.06						
HVD30066	.40	.02	.55	.06						
HVD10029	.41	.03	.20	.08	-.29	.16	.29	.14		
HVD10134	.33	.01	.05	.05	.08	.09	-.08	.08		
HVD10043	.52	.03	.89	.05	.88	.08	-.88	.08		
HVD30060	.71	.07	1.27	.08						
HVD30016	.41	.05	1.85	.16						
HVD30174	.64	.08	3.11	.22						
HVD10119	.73	.04	-1.11	.07						
HVD10033	.85	.04	-.05	.04						
HVD10011	.29	.01	.73	.07	3.13	.11	-3.13	.11		
HVD30047	.47	.03	.96	.09	2.11	.13	-2.11	.14		
HVD30079	.67	.07	-.53	.12						
HVD10106	.49	.04	.86	.07						
HVD30046	.68	.03	.76	.04						
HVD30246	.58	.03	.69	.04						
HVD30346	.89	.04	1.64	.04						
HVD90942	.53	.06	1.70	.12						

Table 16.
Grade 5 Form N: GPCM Item Parameter Estimates

Item ID	Discrimination (a_i)		Difficulty (b_i)		Step parameter for score 1 (d_{i1})		Step parameter for score 2 (d_{i2})		Step parameter for score 3 (d_{i3})	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
HVD10012	.54	.08	-1.65	.25						
HVD30074	.51	.05	-.72	.15						
HVD30032	.46	.03	.54	.07	-.15	.14	.15	.13		
HVD30048	.48	.05	-.74	.16						
HVD30071	.76	.07	.83	.08						
HVD30006	.48	.04	.94	.07						
HVD30306	.57	.05	3.39	.18						
HVD10125	.83	.04	-.76	.05						
HVD10325	.92	.05	-1.10	.06						
HVD30066	.40	.02	.55	.06						
HVD30025	.50	.05	1.38	.12						
HVD30088	.40	.04	1.59	.15						
HVD10203	.32	.03	.50	.11						
HVD10024	.27	.04	1.45	.20						
HVD10134	.33	.01	.05	.05	.08	.09	-.08	.08		
HVD10023	.59	.03	-.38	.04	1.73	.13	-.52	.08	-1.21	.07
HVD10045	.80	.07	1.47	.08						
HVD10086	.73	.07	.97	.08						
HVD30062	.40	.04	.44	.13						
HVD10119	.73	.04	-1.11	.07						
HVD10033	.85	.04	-.05	.04						
HVD10011	.29	.01	.73	.07	3.13	.11	-3.13	.11		
HVD30043	.41	.01	1.23	.04	-.89	.09	.89	.10		
HVD30090	.52	.04	1.04	.07						
HVD10108	.52	.04	1.28	.08						
HVD30046	.68	.03	.76	.04						
HVD30246	.58	.03	.69	.04						
HVD30346	.89	.04	1.64	.04						
HVD30055	.96	.08	.90	.07						

Raw Score to Scale Score Conversion Tables

In creating raw score to scale score conversion tables, it was found that removing two difficult items from each grade 4 form and two easy items from each grade 5 forms would increase scale separation between grades 4 and 5 test forms and improve the face validity of the vertical scale across grades 4 and 5. In addition, shortening the test should help to mitigate the apparent speededness issue of each original test form. Table 17 lists the items removed from each form as well as their item statistics. Note that the criterion for removing items was primarily based on item difficulty parameter estimates.

Table 18 compares the reliabilities (Cronbach's alpha) for each test form with all items included and with two items removed. The reliabilities were virtually the same for each test form; therefore, removing items had a negligible effect on test reliabilities. Table 19 compares the means of item discrimination and difficulty parameter estimates for each test form with all items included and with two items removed. It shows that within-grade test forms were more similar than across-grade test forms in terms of item parameter estimates, as the means of item discrimination and difficulty parameter estimates were closer among within-grade test forms than across-grade test forms. Removing two items from each form did widen the gap of test difficulties between grades 4 and 5 test forms, as evidenced by the fact that the overall mean difference of difficulty parameter estimates between grades 4 and 5 increased from 0.38 with all items included to 0.66 with two items removed from each form. The increasing differences of test difficulties between grades 4 and 5 test forms are also easily seen in the test characteristic curves (TCC; see Figures 4 and 5 for test forms with all items included and with two items removed from each form, respectively). The TCC plots draw scale scores against expected percents correct of IRT true scores for each test form[10]; for a given scale score, an examinee gets higher percent correct of IRT true score for an easy test than for a hard test.

Table 20 shows the raw score to scale score conversion tables for all eight test forms with two items removed from each form. A scoring table includes raw scores, the corresponding IRT scale scores, and their standard errors. Note that scale scores were predefined to have a population mean of 100 and standard deviation of 10 in grade 4.

[10] That percents correct of IRT true scores rather than IRT true scores were used in the TCC plots is to facilitate the comparisons among different test forms because they have different maximum total raw scores (hence, different maximum IRT true scores).

Table 17.
Items Removed in Creating Scoring Tables

Form	First Item Removed					Second Item Removed				
	Harvard ID	Discrimination	Difficulty	P+ value	Max score	Harvard ID	Discrimination	Difficulty	P+ value	Max score
F	HVD90229	.72	1.74	.15	1	HVD30050	.37	1.38	.25	2
G	HVD30081	.22	3.20	.24	1	HVD30063	.57	1.92	.18	1
H	HVD10102	.25	2.55	.30	2	HVD30057	.57	2.41	.12	1
J	HVD30001	.24	2.82	.26	2	HVD10102	.25	2.55	.26	2
K	HVD30031	.60	-.30	.69	1	HVD10131	.80	-.07	.67	1
L	HVD90204	.46	-.86	.73	1	HVD10122	.73	-.52	.72	1
M	HVD30079	.67	-.53	.77	1	HVD10122	.73	-.52	.77	1
N	HVD10012	.54	-1.65	.85	1	HVD30048	.48	-.74	.73	1

Table 18.
Comparison of Reliabilities (Cronbach's Alpha)

Form	All items	Two items removed
F	0.85	0.84
G	0.83	0.83
H	0.82	0.82
J	0.82	0.83
K	0.87	0.86
L	0.89	0.88
M	0.85	0.84
N	0.88	0.88

Table 19.
Comparison of Mean Item Discrimination and Difficulty Parameter Estimates

Grade	Form	All items		Two items removed	
		Discrimination	Difficulty[a]	Discrimination	Difficulty[a]
4	F	.68	.16	.69	.02
	G	.65	.20	.67	.06
	H	.62	.27	.64	.07
	J	.62	.21	.64	-.13
	Total	.64	.21	.66	.01
5	K	.58	.71	.57	.77
	L	.61	.54	.61	.61
	M	.58	.63	.57	.70
	N	.58	.50	.58	.61
	Total	.59	.59	.58	.67

[a] Weighted mean of item difficulty parameter estimates by maximum item scores.

Figure 4. ***Test characteristic curves with all items included.***

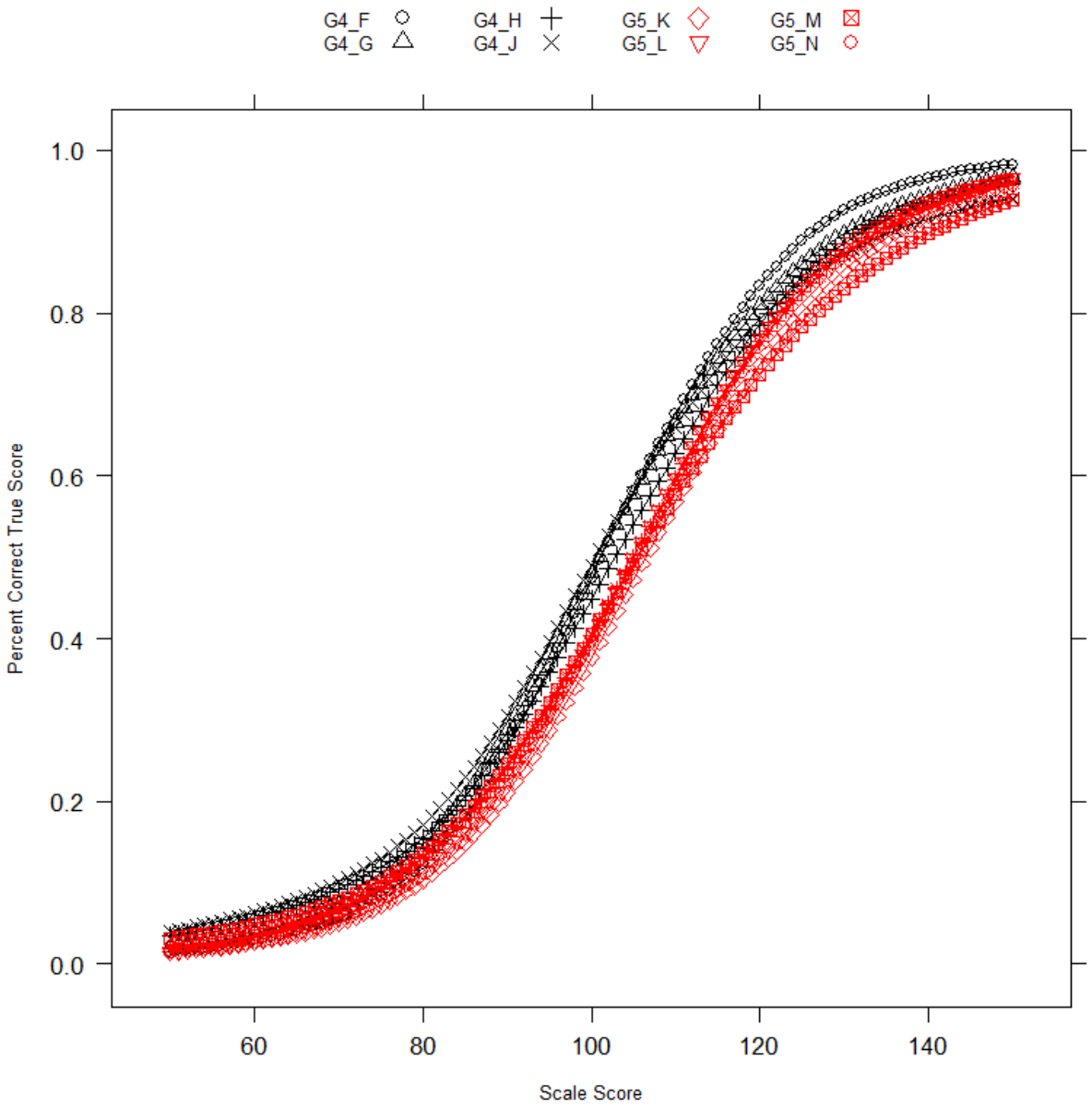


Figure 5. ***Test characteristic curves with two items removed from each test form.***

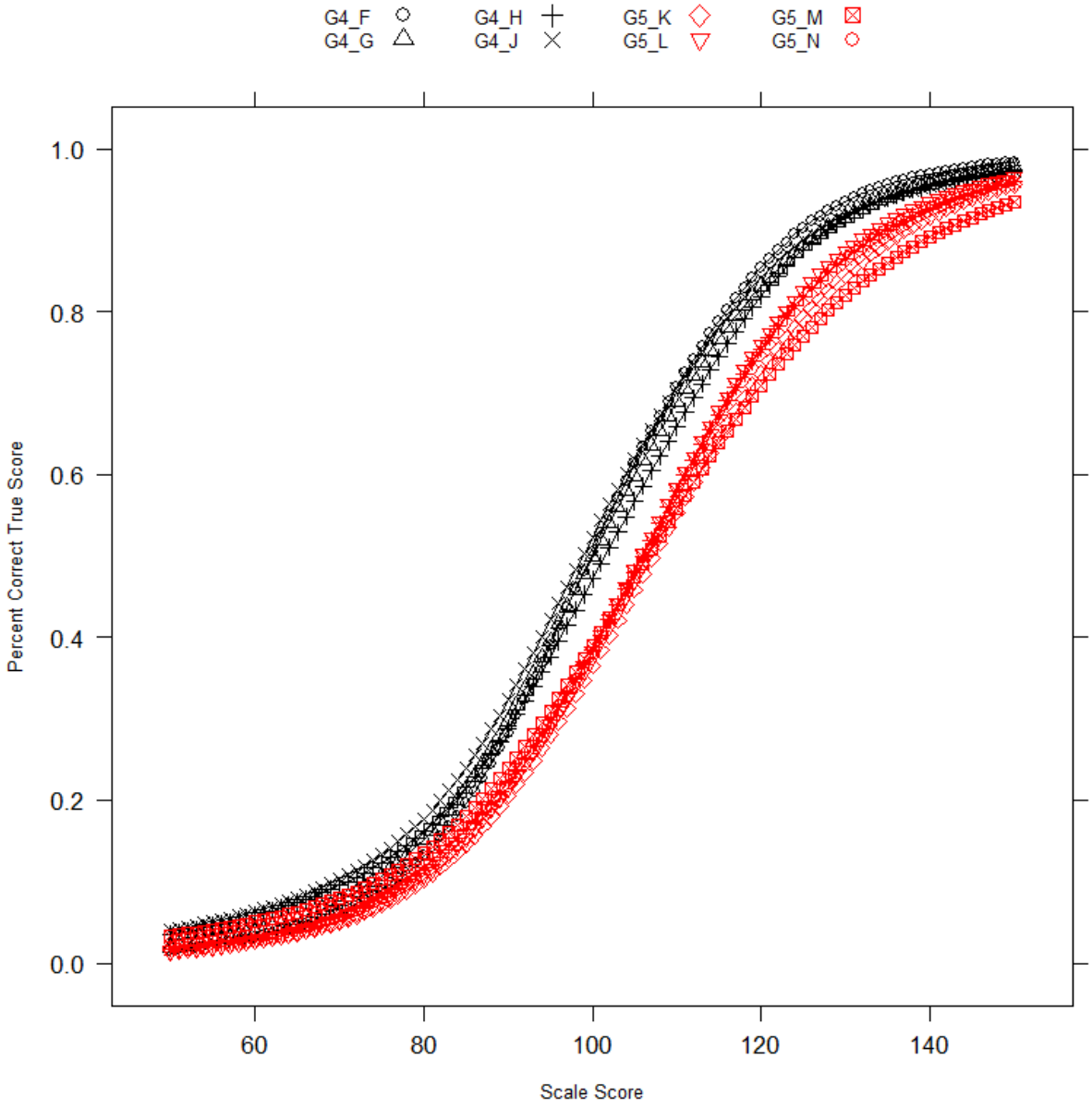


Table 20.

Raw Score to Scale Score Conversion Table with Two Items Removed from Each Form

Raw scores	Form F		Form G		Form H		Form J		Form K		Form L		Form M		Form N	
	Scale score	CSEM	Scale score	CSEM	Scale score	CSEM	Scale score	CSEM	Scale score	CSEM	Scale score	CSEM	Scale score	CSEM	Scale score	CSEM
0	40	24	40	22	40	21	40	22	45	23	45	21	45	22	45	21
1	61	13	57	14	47	18	46	19	62	14	62	13	49	20	59	14
2	71	8	67	9	61	12	62	12	73	9	72	9	66	12	70	9
3	76	7	73	7	70	9	70	9	79	7	78	7	74	8	76	7
4	80	5	77	6	75	7	76	7	83	6	82	6	79	7	80	6
5	83	5	80	5	79	6	80	6	86	5	85	5	83	6	84	6
6	85	4	83	5	83	5	83	5	89	5	87	5	86	5	87	5
7	87	4	85	4	85	5	85	5	91	5	90	4	89	5	89	5
8	89	4	87	4	88	4	87	4	93	4	92	4	91	5	91	5
9	91	4	89	4	90	4	89	4	95	4	94	4	94	5	93	4
10	92	4	91	4	91	4	91	4	97	4	96	4	96	5	95	4
11	94	3	92	4	93	4	93	4	99	4	97	4	98	4	97	4
12	95	3	94	4	95	4	95	4	101	4	99	4	100	4	99	4
13	97	3	96	4	97	4	96	4	102	4	101	4	102	4	101	4
14	98	3	97	4	98	4	98	4	104	4	102	4	104	4	102	4
15	100	3	99	4	100	4	100	4	106	4	104	4	106	4	104	4
16	101	4	100	4	101	4	101	4	107	4	105	4	107	4	105	4
17	103	4	102	4	103	4	103	4	109	4	107	4	109	4	107	4
18	104	4	103	4	105	4	105	4	110	4	108	4	111	5	108	4
19	106	4	105	4	106	4	107	4	112	4	110	4	113	5	110	4
20	108	4	106	4	108	4	109	4	114	4	111	4	115	5	111	4
21	110	4	108	4	110	4	111	4	116	4	113	4	118	5	113	4
22	112	4	110	4	112	4	113	5	118	5	114	4	120	5	115	4
23	114	4	112	4	113	4	116	5	120	5	116	4	123	5	116	4
24	116	5	114	4	115	4	119	5	122	5	118	4	125	6	118	4
25	118	5	116	5	117	4	122	6	124	5	120	5	128	6	120	5
26	121	5	118	5	120	5	126	7	127	6	122	5	132	7	122	5
27	125	6	121	5	122	5	132	9	130	6	124	5	137	8	125	5
28	130	8	124	6	125	6	145	17	134	7	127	6	142	9	128	6
29	139	13	128	7	129	7	160	26	138	8	131	7	150	12	132	7
30	160	24	134	9	134	9			144	10	135	8	164	18	136	8
31			145	15	146	16			155	14	141	10	170	21	143	10
32			160	22	160	25			170	20	153	15			155	15
33											170	23			170	22

Note. CSEM = conditional standard error of measurement.

Validity

The main goal of student assessment development was to produce an instrument that would be sensitive to teachers' MKT. In a study conducted by the National Center for Teacher Effectiveness, value-added scores constructed from these assessments did significantly correlate with teachers' MKT (r=0.15, significant at $p<0.01$, $n=288$). This correlation was higher than between MKT and value-added estimates constructed from state assessments (r=0.06, $p < 0.32$, $n=291$).

Student outcomes, as measured by this assessment, also correlated with other related constructs of interest. The correlation between performance on the NCTE test and the state test, where both tests were taken by our sample in the spring of 2012, is 0.69. The correlation between teacher value-added scores constructed with NCTE and state tests is 0.51. Table 20 shows the relationship between other key predictors and 2012 spring student outcomes on the this test, controlling for fall achievement, their demographic characteristics, peer and school aggregates, and district fixed effects. Each coefficient is entered into the model separately, but combined for presentation.

Regression Coefficients, Controlling for Student Prior Achievement, Demographics, Peer and School aggregates, District fixed effects, and Teacher Clustering
Outcome: Student's 2012 spring test score

Predictors	Regression Coefficient	p-value
2012 State Math Test Score	0.51	0
Student Reports of Efficacy and Perseverance in mathematics	0.097	0
Student Reports of Happiness and Love of Math	0.08	0
Student Reports of Problem Behavior	-0.048	0
Student Score on Bullying Construct	-0.033	0.003
Student's Teacher's Richness	0.04	0.1
Student's Teacher's Working With Student Ideas	0.055	0.002
Student's Teacher's MQI-T Score	0.045	0.006

Outcomes on the spring NCTE test are associated with students' spring score on the state math test; their own reports of efficacy and perseverance in math, and their love of math. Student performance in spring is negatively related to reports of problem behavior and bullying within their classroom. Student performance on the spring NCTE test is also predicted by an observational measure of the extent to which teachers work with student ideas, and an observational measure of the mathematical quality of instruction (MQI-T). Teacher richness, in the sense that instruction is focused on mathematical meaning or practices, is marginally significantly related to students' spring outcomes on the NCTE test.

References

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